

# Symmetries

Henning Stahlberg,  
Biozentrum, Uni Basel, Switzerland  
c-cina.org

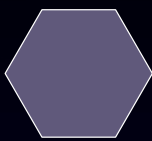
2dx Workshop  
Basel, August 23-26, 2016

1

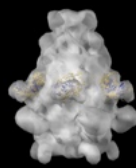
7-2dx-2016-Symmetries.key - 22 Aug 2016

## Molecular symmetry

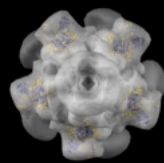
Cyclic symmetry



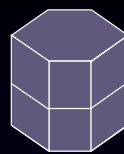
C6



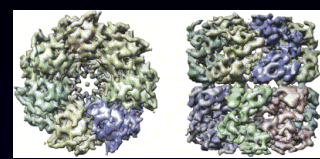
C5 supercomplex



Dihedral symmetry



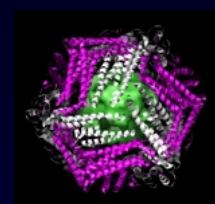
D6



D7 - GroEL

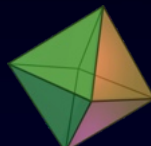
Ludtke et al. *Structure* (2004) 12:1129-36

Tetrahedral symmetry



Insect Ferritin

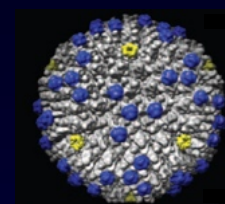
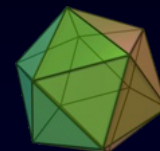
Octahedral symmetry



Hsp16.5

Kim et al. *Nature* (1998) 394:595-99

Icosahedral symmetry



Virus

Slide by Michael Landsberg, Brisbane, Australia

2

7-2dx-2016-Symmetries.key - 22 Aug 2016

## Plane group notation

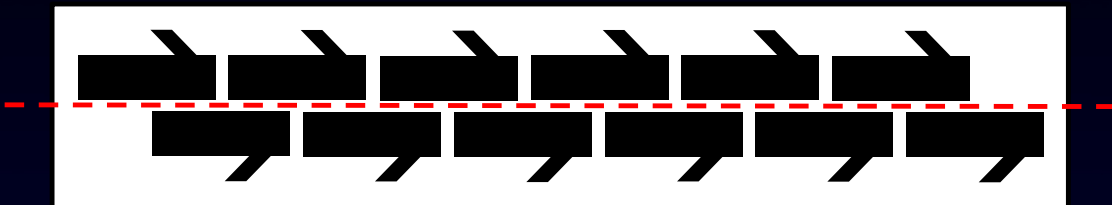
### Hermann-Mauguin style

- Begins with either  $p$  or  $c$ , for a primitive cell or a face-centered cell
- This is followed by a digit,  $n$ , indicating the highest order of rotational symmetry: 1-fold (none), 2-fold, 3-fold, 4-fold, or 6-fold
- The next two symbols indicate symmetries relative to the "main" translation axis of the pattern; if there is a mirror perpendicular to a translation axis this is the main one (or if there are two, one of them).
  - ⇒ The symbols are either  $m$ ,  $g$ , or  $1$ , for mirror, glide reflection, or none.
  - ⇒ The axis of the mirror or glide reflection is perpendicular to the main axis for the first letter...
  - ⇒ ...and either parallel or tilted  $180^\circ/n$  (when  $n > 2$ ) for the second letter.

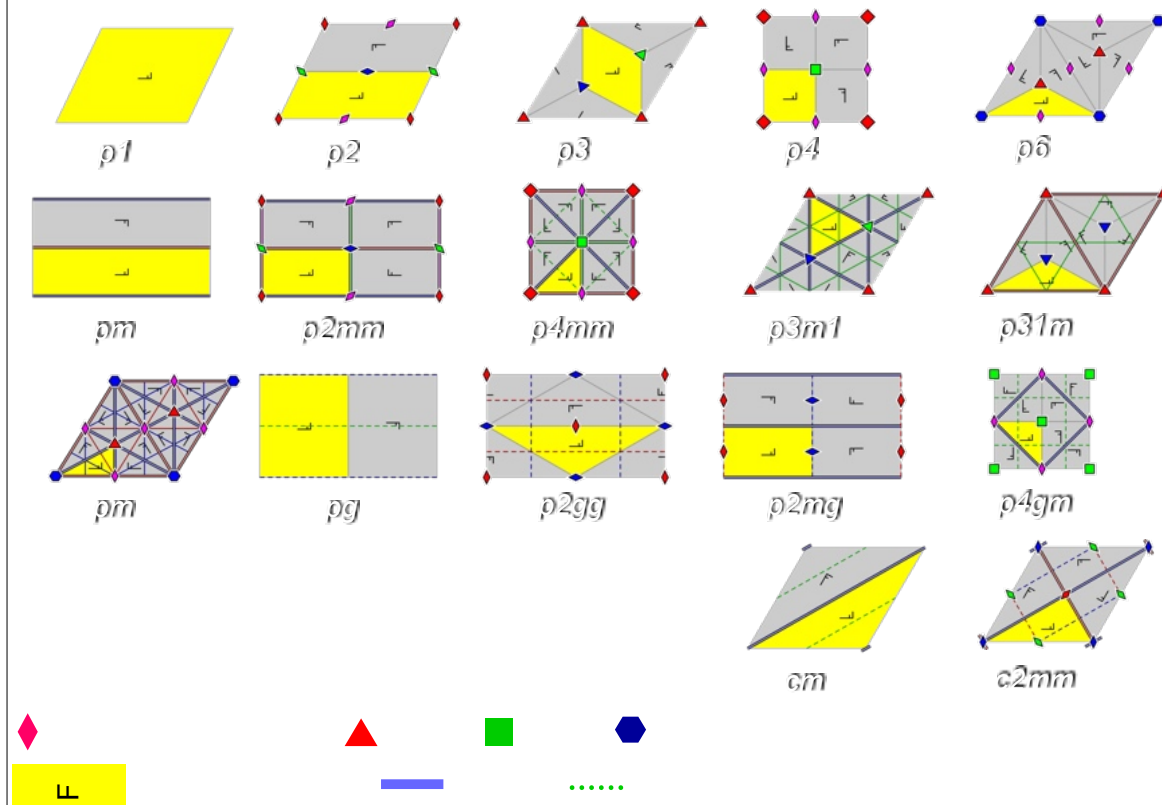
*p2mg*

## Translation + Reflection = Glide Reflection

Translation by  $\frac{1}{2}$  unit cell combined with a reflection about a line parallel to the direction of translation



## Plane group definitions



5

7-2dx-2016-Symmetries.key - 22 Aug 2016

## Group theory

- A crystallographic **space group** is the mathematical group of symmetry operations which apply to both the given unit cell and the crystal array
- There are **230** possible crystallographic space groups in 3D
  - **65** for proteins and chiral molecules
- **17 plane groups** describe all the possible symmetry arrangements in projection images of 2D crystals
- These plane groups are different (but correlate somewhat trivially) to the **17 2D space groups** which describe all possible 2D crystal arrangements

6

7-2dx-2016-Symmetries.key - 22 Aug 2016

## Symmetry of 2D crystals

Two-sided Plane Group*	Corresponding 3-dimensional space group	Projection Symmetry
p1	P1	p1
p21	P2(c-axis unique)	p2
p12	P2(b-axis unique)	pm
p12 <sub>1</sub>	P2 <sub>1</sub>	pg
c12 <sub>1</sub>	C2 <sub>1</sub>	cm
p222	P222	pmm
p222 <sub>1</sub>	P222 <sub>1</sub>	pmg
p22 <sub>1</sub> <sup>2</sup> <sub>1</sub>	P22 <sub>1</sub> <sup>2</sup> <sub>1</sub>	pgg
c222 <sub>1</sub>	C222 <sub>1</sub>	cmm
p4	P4	p4
p422	P422	p4m
p42 <sub>1</sub> <sup>2</sup>	P42 <sub>1</sub> <sup>2</sup>	p4g
p3	P3	p3
p312	P312	p3m1
p321	P321	p31m
p6	P6	p6
p622	P622	p6m

\* We use here the nomenclature for the two-sided plane groups proposed by Holser (1958). This nomenclature has the following main rules.

- Cell type is indicated first by a small letter, p or c.
- The axis perpendicular to the plane is always chosen as the z-axis.
- The symmetry along this axis is always described by the first symbol following the cell type.

† There are 80 two-sided plane groups in total, of which the 17 given here are the only ones not containing inversion centres or mirror or glide planes.

## Symmetry of 2D crystals

- An electron micrograph is a projection of the object along the direction of the electron beam.
- Two-dimensional plane groups describe the projection symmetry, 17 possibilities. Listed in the Int Tab for X-ray Crystallography.
- The specimen has an extension in the third dimension ("top and bottom").
- Two-sided plane groups describe the symmetry of 2D crystals, 17 possibilities for biological molecules.
- The smallest repeating unit, the unit cell is described by the lattice vectors a and b and the angle gamma between them.
- Depending on the symmetry, the unit cell may contain multiple copies of the molecule.
- In addition, non-crystallographic symmetry may add even more molecules to the unit cell.
- The packing arrangement in a 2D crystal can give information about oligomeric arrangement of the protein.



# Symmetry of 2D crystals

**pg** No. 4

Origin on g Co-ordinates:  $|x, y|$

$$A-2 \cos 2\pi \left( hx - \frac{k}{4} \right) \cos 2\pi \left( ky - \frac{k}{4} \right)$$

$$B-2 \cos 2\pi \left( hx - \frac{k}{4} \right) \sin 2\pi \left( ky - \frac{k}{4} \right)$$

$k-2n$   $A-2 \cos 2\pi hx \cos 2\pi ky$   
 $B-2 \cos 2\pi hx \sin 2\pi ky$   $-0$  if  $k=0$

$k-2n+1$   $A-2 \sin 2\pi hx \sin 2\pi ky$   
 $B-2 \sin 2\pi hx \cos 2\pi ky$   $A-B=0$  if  $h=0$

$$\rho(XY) = \frac{1}{A_L} \left[ F(00) + 2 \left[ \sum_{k=1}^{h-2n} |F(hk)| \cos 2\pi hX + \sum_{k=1}^{h-2n} |F(hk)| \cos [2\pi kY - \alpha(hk)] \right] + 4 \left[ \sum_{k=1}^{h-2n} |F(hk)| \cos 2\pi hX \cos [2\pi kY - \alpha(hk)] - \sum_{k=1}^{h-2n} |F(hk)| \sin 2\pi hX \sin [2\pi kY - \alpha(hk)] \right] \right]$$

**pmg** No. 7

Origin at 2 Co-ordinates:  $\pm |x, y|$

$$A-4 \cos 2\pi \left( hx + \frac{k}{4} \right) \cos 2\pi \left( ky - \frac{k}{4} \right)$$

$$B=0$$

$h-2n$   $A-4 \cos 2\pi hx \cos 2\pi ky$ ;  $B=0$

$h-2n+1$   $A-4 \sin 2\pi hx \sin 2\pi ky$ ;  $A-B=0$  if  $k=0$

$$\rho(XY) = \frac{1}{A_L} \left[ F(00) + 2 \left[ \sum_{k=1}^{h-2n} |F(hk)| \cos 2\pi hX + \sum_{k=1}^{h-2n} |F(hk)| \cos 2\pi kY \right] + 4 \left[ \sum_{k=1}^{h-2n} |F(hk)| \cos 2\pi hX \cos 2\pi kY - \sum_{k=1}^{h-2n} |F(hk)| \sin 2\pi hX \sin 2\pi kY \right] \right]$$

---

**cm** No. 5

Origin on m Co-ordinates:  $(0,0; \frac{1}{2}, \frac{1}{2}) + |x, y|$

$$A-4 \cos^2 2\pi \left( hx + \frac{k}{4} \right) \cos 2\pi hx \cos 2\pi ky$$

$$B-4 \cos^2 2\pi \left( hx + \frac{k}{4} \right) \cos 2\pi hx \sin 2\pi ky$$

$h+k-2n$   $A-4 \cos 2\pi hx \cos 2\pi ky$   
 $B-4 \cos 2\pi hx \sin 2\pi ky$   $-0$  if  $k=0$

$h+k-2n+1$   $A-B=0$

$$\rho(XY) = \frac{1}{A_L} \left[ F(00) + 2 \left[ \sum_{k=1}^{h-2n} |F(hk)| \cos 2\pi hX + \sum_{k=1}^{h-2n} |F(hk)| \cos [2\pi kY - \alpha(hk)] \right] + 4 \sum_{k=1}^{h-2n} |F(hk)| \cos 2\pi hX \cos [2\pi kY - \alpha(hk)] \right]$$

**pgg** No. 8

Origin at 2 Co-ordinates:  $\pm |x, y|$

$$A-4 \cos 2\pi \left( hx + \frac{k}{4} \right) \cos 2\pi \left( ky - \frac{k}{4} \right)$$

$$B=0$$

$h+k-2n$   $A-4 \cos 2\pi hx \cos 2\pi ky$ ;  $B=0$

$h+k-2n+1$   $A-4 \sin 2\pi hx \sin 2\pi ky$   
 $A-B=0$  if  $h=0$  or  $k=0$

$$\rho(XY) = \frac{1}{A_L} \left[ F(00) + 2 \left[ \sum_{k=1}^{h-2n} |F(hk)| \cos 2\pi hX + \sum_{k=1}^{h-2n} |F(hk)| \cos 2\pi kY \right] + 4 \left[ \sum_{k=1}^{h-2n} |F(hk)| \cos 2\pi hX \cos 2\pi kY - \sum_{k=1}^{h-2n} |F(hk)| \sin 2\pi hX \sin 2\pi kY \right] \right]$$

---

**pmm** No. 6

Origin at 2mm Co-ordinates:  $\pm |x, y|$

$$A-4 \cos 2\pi hx \cos 2\pi ky$$

$$B=0$$

$$\rho(XY) = \frac{1}{A_L} \left[ F(00) + 2 \left[ \sum_{k=1}^{h-2n} |F(hk)| \cos 2\pi hX + \sum_{k=1}^{h-2n} |F(hk)| \cos 2\pi kY \right] + 4 \sum_{k=1}^{h-2n} |F(hk)| \cos 2\pi hX \cos 2\pi kY \right]$$

**cm** No. 9

Origin at 2mm Co-ordinates:  $(0,0; \frac{1}{2}, \frac{1}{2}) \pm |x, y|$

$$A-8 \cos^2 2\pi \left( hx + \frac{k}{4} \right) \cos 2\pi hx \cos 2\pi ky$$

$$B=0$$

$h+k-2n$   $A-8 \cos 2\pi hx \cos 2\pi ky$ ;  $B=0$

$h+k-2n+1$   $A-B=0$

$$\rho(XY) = \frac{1}{A_L} \left[ F(00) + 2 \left[ \sum_{k=1}^{h-2n} |F(hk)| \cos 2\pi hX + \sum_{k=1}^{h-2n} |F(hk)| \cos 2\pi kY \right] + 4 \sum_{k=1}^{h-2n} |F(hk)| \cos 2\pi hX \cos 2\pi kY \right]$$

**Get Spacegroup & PhaseOrigin**

Output (Double click for logbrowser) Verboosity Level: Low

```

==== Anaconda Python executable not found. Is it installed ??? =====
/opt/anaconda/bin/python
==== The location of Anaconda Python is defined in Preferences. =====

Symmetries to test = ALL
Stepsize and Phase Search Array Size = 2, 181
IQ Max = 3, Resolution Max = 9.0
    
```

SPACEGROUP	Pha.Res. (#) v.other spots (90 random)	Pha.Res. (#) v.theoretical (45 random)	OX	OY	TX	TY	Target
1 p1	16.6	396	11.9	396			
2 p2	40.6	198	20.3	396	-379.5	36.1	23.9
3b p12_b	80.3	163	45.5	10	-449.8	-49.7	16.9
3a p12_a	82.5	165	9.4	14	-173.5	36.3	17.0
4b p121_b	16.7	163	13.0	10	-289.6	0.3	16.9
4a p121_a	35.5	165	20.3	14	-429.5	-53.8	17.0
5b c12_b	80.3	163	45.5	10	-449.8	-49.7	16.9
5a c12_a	82.5	165	9.4	14	-173.5	36.3	17.0
6 p222	68.0	526	20.3	396	-379.6	-143.8	19.4
7b p2221b	66.6	526	40.5	396	-359.3	-53.8	19.4
7a p2221a	61.8	526	40.9	396	-289.6	126.5	19.4
8 p22121	31.6	526	20.3	396	-379.6	-143.8	19.4
9 c222	68.0	526	20.3	396	-379.6	-143.8	19.4
10 p4	33.1	506	20.3	396	-379.5	36.0	19.5
11 p422	61.3	1154	20.3	396	-199.7	36.2	17.9
12 p4212	30.1	1154	20.3	396	-379.6	-143.9	17.9
13 p3	68.6	192	--	--	-138.0	-23.5	16.6
14 p312	60.3	539	16.6	30	-258.5	95.2	16.8
15 p321	56.6	539	17.2	30	-109.9	125.8	16.8
16 p6	60.4	582	20.4	396	-379.1	-143.8	19.1
17 p622	56.7	1276	20.3	396	-379.3	-143.9	17.8

OX,OY = best phase origin for this symmetry  
TX,TY = best beam tilt for this symmetry  
Target = target resid. based on statistics, taking Friedel weight into account

==== Best SpaceGroup is p121\_b =====  
==== 2dx\_allspace - normal end. =====  
#####2dx\_allspace finished.#####

Status

Calc. Mag#	49743.9						Last QVAL	2025.2
Unbend	IQ1	IQ2	IQ3	IQ4	IQ5	IQ6	QVAL	
Unb.I	54	71	63	53	40	24	1674.0	
Unb.II	62	94	77	50	27	25	2025.2	
MovieA	--	--	--	--	--	--	--	
MovieB	--	--	--	--	--	--	--	
PowerBliss			[A]	15	12	9	7	
(noIse=7)			#	654	58	117	22	
Axis Info			Defoc.		Letf.		SpSplit	
Grid TAxis			---		---		Merge	
Grid TAngle			---		-9.8		20.0	
Xst. TAxis			---		-8.5		-1.4	
Xst. TAngle			---		-9.8		-1.4	

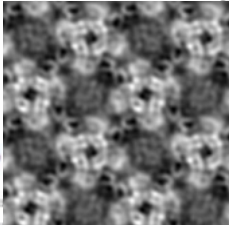
Symmetries to test = ALL  
 Stepsize and Phase Search Array Size = 2 , 181  
 IQ Max = 3, Resolution Max = 9.0

SPACEGROUP	Phs.Res. (#) v.other spots (90 random)	Phs.Res. (#) v.theoretical (45 random)	OX	OY	TX	TY	Target
1 p1	16.6 396	11.9 396					
2 p2	40.6 198	20.3 396	-379.5	36.1	0.00	0.00	23.9
3b p12_b	80.3 163	45.5 10	-449.8	-49.7	0.00	0.00	16.9
3a p12_a	82.5 165	9.4 14	-173.5	36.3	0.00	0.00	17.0
4b p121_b	16.7* 163	13.0 10	-289.6	0.3	0.00	0.00	16.9
4a p121_a	35.5 165	20.3 14	-429.5	-53.8	0.00	0.00	17.0
5b c12_b	80.3 163	45.5 10	-449.8	-49.7	0.00	0.00	16.9
5a c12_a	82.5 165	9.4 14	-173.5	36.3	0.00	0.00	17.0
6 p222	68.0 526	20.3 396	-379.6	-143.8	0.00	0.00	19.4
7b p2221b	66.6 526	40.5 396	-359.3	-53.8	0.00	0.00	19.4
7a p2221a	61.8 526	40.9 396	-289.6	126.5	0.00	0.00	19.4
8 p22121	31.6 526	20.3 396	-379.6	-143.8	0.00	0.00	19.4
9 c222	68.0 526	20.3 396	-379.6	-143.8	0.00	0.00	19.4
10 p4	33.1 506	20.3 396	-379.5	36.0	0.00	0.00	19.5
11 p422	61.3 1154	20.3 396	-199.7	36.2	0.00	0.00	17.9
12 p4212	30.1 1154	20.3 396	-379.6	-143.9	0.00	0.00	17.9
13 p3	68.6 192	-- --	-138.0	-23.5	0.00	0.00	16.6
14 p312	60.3 539	16.6 30	-258.5	95.2	0.00	0.00	16.8
15 p321	56.6 539	17.2 30	-109.9	125.8	0.00	0.00	16.8
16 p6	60.4 582	20.4 396	-379.1	-143.8	0.00	0.00	19.1
17 p622	56.7 1276	20.3 396	-379.3	-143.9	0.00	0.00	17.8

\* = acceptable  
 ! = should be considered  
 ~ = possibility

OX,OY = best phase origin for this symmetry  
 TX,TY = best beam tilt for this symmetry  
 Target = target resid. based on statistics, taking Friedel weight into

==== Best SpaceGroup is p121\_b =====



Symmetries to test = ALL  
 Stepsize and Phase Search Array Size = 2 , 181  
 IQ Max = 3, Resolution Max = 9.0

SPACEGROUP	Phs.Res. (#) v.other spots (90 random)	Phs.Res. (#) v.theoretical (45 random)	OX	OY	TX	TY	Target
1 p1	16.6 396	11.9 396					
2 p2	40.6 198	20.3 396	-379.5	36.1	0.00	0.00	23.9
3b p12_b	80.3 163	45.5 10	-449.8	-49.7	0.00	0.00	16.9
3a p12_a	82.5 165	9.4 14	-173.5	36.3	0.00	0.00	17.0
4b p121_b	16.7* 163	13.0 10	-289.6	0.3	0.00	0.00	16.9
4a p121_a	35.5 165	20.3 14	-429.5	-53.8	0.00	0.00	17.0
5b c12_b	80.3 163	45.5 10	-449.8	-49.7	0.00	0.00	16.9
5a c12_a	82.5 165	9.4 14	-173.5	36.3	0.00	0.00	17.0
6 p222	68.0 526	20.3 396	-379.6	-143.8	0.00	0.00	19.4
7b p2221b	66.6 526	40.5 396	-359.3	-53.8	0.00	0.00	19.4
7a p2221a	61.8 526	40.9 396	-289.6	126.5	0.00	0.00	19.4
8 p22121	31.6 526	20.3 396	-379.6	-143.8	0.00	0.00	19.4
9 c222	68.0 526	20.3 396	-379.6	-143.8	0.00	0.00	19.4
10 p4	33.1 506	20.3 396	-379.5	36.0	0.00	0.00	19.5
11 p422	61.3 1154	20.3 396	-199.7	36.2	0.00	0.00	17.9
12 p4212	30.1 1154	20.3 396	-379.6	-143.9	0.00	0.00	17.9
13 p3	68.6 192	-- --	-138.0	-23.5	0.00	0.00	16.6
14 p312	60.3 539	16.6 30	-258.5	95.2	0.00	0.00	16.8
15 p321	56.6 539	17.2 30	-109.9	125.8	0.00	0.00	16.8
16 p6	60.4 582	20.4 396	-379.1	-143.8	0.00	0.00	19.1
17 p622	56.7 1276	20.3 396	-379.3	-143.9	0.00	0.00	17.8

**Why 21 space groups?**

\* = acceptable  
 ! = should be considered  
 ~ = possibility

OX,OY = best phase origin for this symmetry  
 TX,TY = best beam tilt for this symmetry  
 Target = target resid. based on statistics, taking Friedel weight into

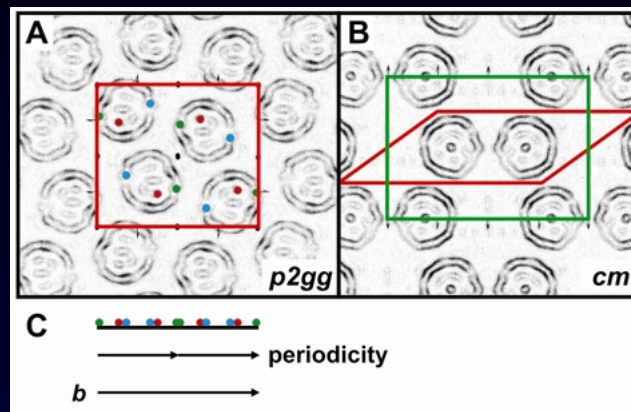
==== Best SpaceGroup is p121\_b =====





## Systematic absences

- **Symmetry forbidden reflections** result when a crystal has periodicity over less than one unit cell
- Axial/zonal systematic absences arise from glides/screws
  - > Loss of odd reflections
- Integral systematic absences arise when a centered cell is chosen
  - > Twice as many reflections



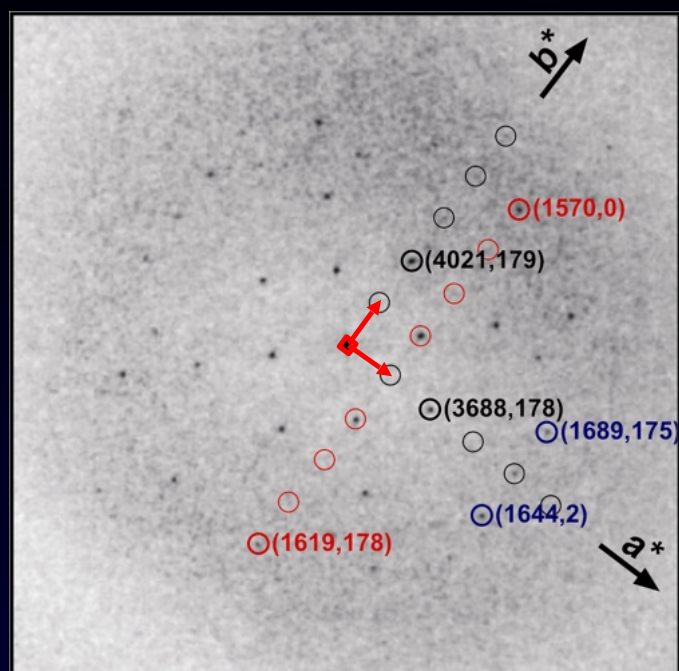
Slide by Michael Landsberg, Brisbane, Australia

13

7-2dx-2016-Symmetries.key - 22 Aug 2016

## Fourier space example – CHIP28 (Aqp1)

- $a=b, \gamma=90$ 
  - square unit cell
- all phases =  $0/180$ 
  - centrosymmetric space group
- points related by 4-fold rotation are equal
  - base symmetry is  $p4$
- odd reflections absent
- spots equidistant from  $a^*$  are out of phase by  $180$ 
  - glide symmetry



Slide by Michael Landsberg, Brisbane, Australia

14

7-2dx-2016-Symmetries.key - 22 Aug 2016

## CHIP28 (Aqp1) in real space

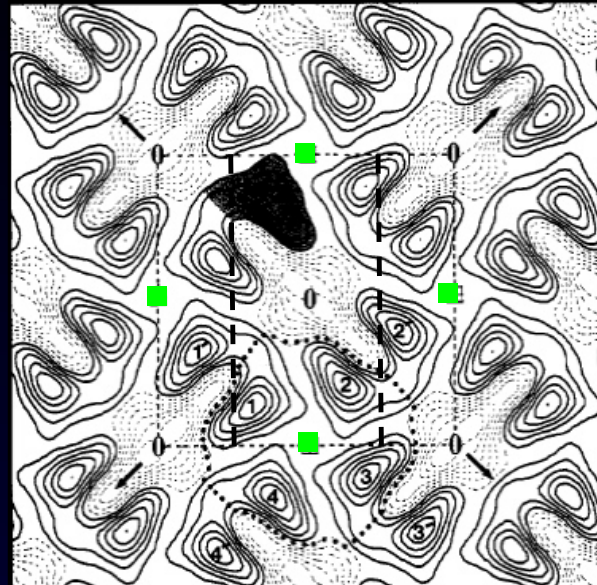
### $p4gm$ ( $P42_12$ )

- 4 fold rotational symmetry
- 1 pair of glide axes
- 1 pair of mirror lines

■ 4-fold center of rotation

- - - glide axis

→ mirror line



Mitra et al. *Biochem* (1994) 33:12735-40

Slide by Michael Landsberg, Brisbane, Australia

15

7-2dx-2016-Symmetries.key - 22 Aug 2016

## Other considerations

- When might symmetry fall apart?
  - Plane group symmetry rules only hold for untilted specimens
  - Astigmatism causes a non-uniform effect of the CTF on symmetry related spots, potentially making symmetry evaluation unreliable
  - Symmetry rules only hold when data are shifted to phase origin
  - Stain exclusion patterns can cause over-estimation of symmetry
  - Low resolution data may also over-estimate symmetry
- Always check for the satisfaction of sub-symmetries to help
- *ALLSPACE* does not check for systematic absences
- Check that symmetry rules continue to hold when merging and moving up in resolution

Slide by Michael Landsberg, Brisbane, Australia

16

7-2dx-2016-Symmetries.key - 22 Aug 2016



Documentation - 2DX Software — 2dx - Image Processing for 2D Crystals

home download documentation workshop contact Log In Register

2D Electron Crystallography Image Processing Suite

you are here: Home > Documentation > 2dx Software

## Documentation - 2DX Software

### 2dx Software

- 2dx User Manual (.pdf)
- Manual
- Screen Shots
- Movie Tutorials
- Known Bugs
- Publications
- Parameters
- 2dx Movies
- Version Change Log

### Sample Prep Movies

### MRC Software Manual A (.pdf) manual of the 2DX software is available [here](#). Please refer to this manual for installation and other details on how to use the software. Frequently Asked Questions A list of frequently asked questions is compiled and can be accessed [here](#). Please use this list to find your question, and if you do not find any help, please contact us [here](#). Video Tutorials Video tutorials are available [here](#). Also refer to the [workshops](#) for more details.

17

7-2dx-2016-Symmetries.key - 22 Aug 2016

GitHub, Inc.

Home - C-CINA/2dx Wiki

This repository Search Pull requests Issues Gist

C-CINA / 2dx Unwatch 8 Star 6 Fork 1

Code Issues 18 Pull requests 0 Wiki Pulse Graphs Settings

## Home

Nikhil Biyani edited this page 4 days ago · 24 revisions

Welcome to the 2dx developer wiki!

## 2dx Developer zone

To contribute to the source code just sign up to GitHub and contact the developers under [nikhil.biyani@unibas.ch](mailto:nikhil.biyani@unibas.ch) or submit an issue to the bug-tracker.

## User Guides

- Beginners User Guide
- Crystallographic symmetries in 2dx

## Advanced User Guides

- Motion Correction GUI
- 2dx\_automator User Documentation
- Move image to second project

## Developer Guides

Pages 19

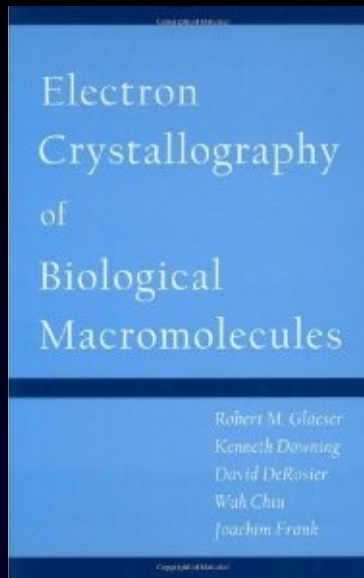
Find a Page...

- Home
- 2dx Development Process
- 2dx Source Code Structure
- 2dx\_automator
- Automated Drift Correction GUI
- Automatic Drift Correction C CINA setup
- Chimera and time series!
- Continuous Integration and packaging with Jenkins
- Crystallographic symmetries in 2dx
- Export image to second project

18

7-2dx-2016-Symmetries.key - 22 Aug 2016

## Further Reading



Available online at [www.sciencedirect.com](http://www.sciencedirect.com)

 **ScienceDirect**

Journal of Structural Biology 160 (2007) 332–343

**Journal of Structural Biology**

[www.elsevier.com/locate/jysbi](http://www.elsevier.com/locate/jysbi)

**Symmetry: A guide to its application in 2D electron crystallography**

Michael J. Landsberg \*, Ben Hankamer \*

*Institute for Molecular Bioscience, Queensland Biosciences Precinct, The University of Queensland, Brisbane, Qld 4072, Australia*

Received 1 May 2007; received in revised form 19 June 2007; accepted 6 July 2007  
Available online 17 July 2007

...also: V. Unger *et al.* "Structure determination from electron micrographs of 2d crystals"

Slide by Michael Landsberg, Brisbane, Australia